

Reg. No. :

Name :

SECOND SEMESTER B.TECH. DEGREE EXAMINATION, MAY/JUNE 2016

MA 102 : DIFFERENTIAL EQUATIONS

Max. Marks : 100

Duration : 3 Hours

PART – A

Answer **all** questions and **each** question carries **3** marks.



1. Determine a linearly independent solution of the differential equation $(x^2 + 1) y'' - 2xy' + 2y = 0$ if $y_1 = x$ is solution.
2. Solve the differential equation $y^{IV} + 6y''' + 9y'' = 0$.
3. Find the particular integral of the differential equation $(D^2 - 2D + 1)y = xe^x$.
4. Solve by the method of variation parameters, $(D^2 + 4)y = \tan 2x$.
5. Develop the Fourier series of $f(x) = x^2$ in $-2 \leq x \leq 2$.
6. Find the Fourier sine series of $f(x) = e^x$ in $0 < x < 1$.
7. Obtain the partial differential equation by eliminating f and g from $z = xf(y) + yg(x)$.
8. Solve the partial differential equation $(y^2 + z^2)p - xyq + xz = 0$.
9. Obtain the solution of the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$ using method of separation of variables when the separation constant $k < 0$.
10. Write any two assumptions involved in deriving one dimensional wave equation.
11. Find the steady state temperature distribution in a rod of length 20 cm if the ends of the rod are kept at 10°C and 70°C .
12. Solve $\frac{\partial u}{\partial t} = h \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0, t) = u(1, t) = 0$ for $t > 0$ and $u(x, 0) = 3 \sin n\pi x$, $0 < x < 1$.

(12x3=36 Marks)

P.T.O.



PART – B

Answer **six** questions – **one full** question from **each** Module.

Module – 1

13. a) Reduce to first order and hence solve the ODE
 i) $y'' + (y')^3 \cos y = 0$ and
 ii) $2xy'' = 3y'$.
- b) Solve the IVP $y'' - 2y' + 5y = 0$, $y(0) = -3$, $y'(0) = 1$. 11
- OR
14. a) Show that the functions x and $x \ln(x)$ are linearly independent (use Wronskian). Hence form an ODE for the given basis x , $x \ln(x)$.
- b) Solve the IV $Py'' + 0.2y' + 4.01y = 0$, $y(0) = 0$, $y'(0) = 2$. 11

Module – 2

15. a) Solve the differential equation $(D + 1)^2y = x^2e^x$.
- b) Solve the differential equation $(x^3D^3 + 3x^2D^2 + xD + 1)y = x + \log x$. 11
- OR
16. a) Solve the differential equation $(D^2 + 1)y = x^2e^x + \sin x$.
- b) Solve the differential equation $(x + 1)^2y'' + (x + 1)y' - y = 2 \sin \log(x + 1)$. 11

Module – 3

17. a) Find the Fourier Series of $f(x) = \begin{cases} x & , 0 < x < 1 \\ 1-x & , 1 < x < 2 \end{cases}$.
- b) Find the Fourier cosine series of $f(x) = x(\pi - x)$ in $0 < x < \pi$. 11
- OR
18. a) Expand $f(x) = e^{-x}$ in $(-l, l)$ as a Fourier Series.
- b) Find the half range sine series of $f(x) = x \sin x$ in $0 < x < \pi$. 11

**Module - 4**

19. a) Form the PDE by eliminating a, b, c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

b) Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2 \frac{\partial^2 z}{\partial y^2} = e^{2x+y}$

OR

20. a) Solve : $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$.

b) Solve the partial differential equation $\frac{\partial^3 z}{\partial x^3} - 4 \frac{\partial^3 z}{\partial^2 x \partial y} + 4 \frac{\partial^3 z}{\partial x \partial y^2} = \cos(2x + y)$. 11

**Module - 5**

21. A tightly stretched string of length 'a' with fixed ends is initially in equilibrium position. Find the displacement $u(x, t)$ of the string if it is set vibrating by giving each of its points a velocity $v_0 \sin(\pi x/a)$. 10

OR

22. A transversely vibrating string of length 'a' is stretched between two points A and B. The initial displacement of each point of the string is zero and the initial velocity at a distance x from A is $kx(a - x)$. Find the form of the string at any subsequent time. 10

Module - 6

23. Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature zero if the initial temperature is $f(x) = \begin{cases} x & , 0 < x < L/2 \\ L-x & , L/2 < x < L \end{cases}$. 10

OR

24. An insulated rod of length L has its ends A and B maintained at 0°C and 100°C respectively until steady state conditions prevails. If B is suddenly reduced to 0°C and maintained at 0°C , then find the temperature in the rod at a distance x from A at time t. 10